



WHEATLANDS PRIMARY SCHOOL

Mathematics: Written Calculation Methods

Produced: April 2021

Reviewed: September 2025

Review: July 2026

Written Calculation Progression

This document maps the required written formal calculation methods as outlined in the National Curriculum (2013) Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division.

This Written Calculation Progression links the key concrete experiences with pictorial and abstract representations (written symbolic and spoken). This supports pupils to move with confidence and deep conceptual understanding through each strand of calculation.

The Importance of Mental Mathematics:

While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies (*see separate document*) and known facts that form the basis of all calculations. Pupils are provided with frequent opportunities to compare and evaluate different calculation strategies. This helps them develop an understanding that efficiency is personal and based on the numbers involved.

Concrete, Pictorial and Abstract:

❖ **Concrete**

Concrete manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways.

❖ **Pictorial**

The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives.

❖ **Abstract - Written**

The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

❖ **Abstract - Spoken**

Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent. This can be scaffolded effectively using speaking frames.

YEAR 2

NC Statement:

Add and Subtract numbers using concrete objects, pictorial representations, and mentally including:

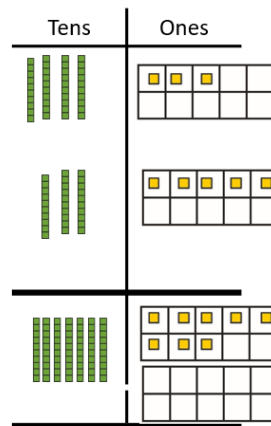
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers.



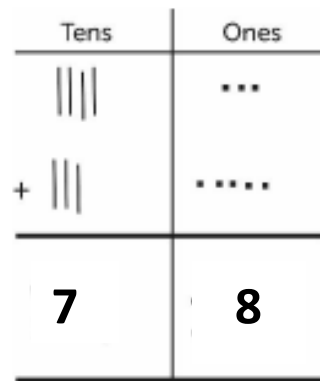
ADDITION

Expanded written method with no exchanging (two-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 40 \quad 3 \\ + 30 \quad 5 \\ \hline 70 \quad 8 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ The sum of ... tens and ... tens is ... tens.
- ❖ So, ... + ... is equal to ... tens and ... ones, which is ...

Notes:

Using embedded tens frame supports pupils to organise ones in preparation for exchanging.

YEAR 2

NC Statement:

Add and Subtract numbers using concrete objects, pictorial representations, and mentally including:

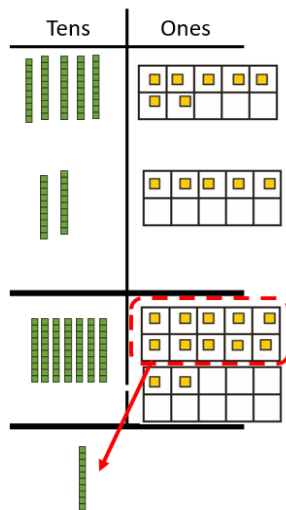
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers.



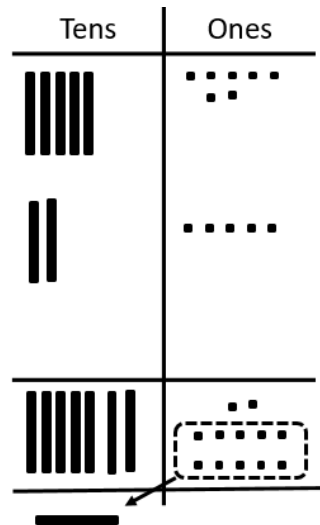
ADDITION

Expanded written method with exchanging of ones (two-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r}
 50 + 7 \\
 + 20 + 5 \\
 \hline
 80 + 12 \\
 \hline
 10
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ This is exchanged into ... ten and ... ones.
- ❖ The sum of ... tens and ... tens is ...tens.
- ❖ So, ... + ... is equal to ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Using embedded tens frame supports pupils to rapidly see the exchange and to keep their jottings organised.

YEAR 3

NC Statement:

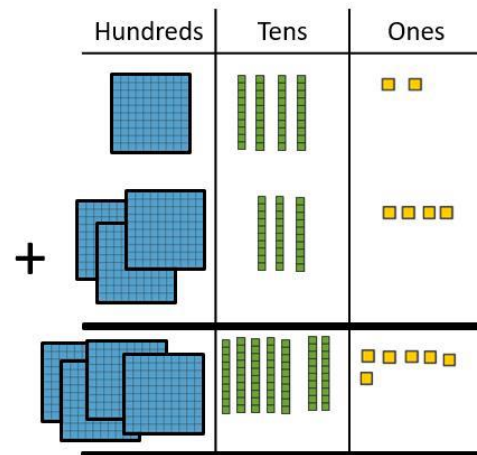
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



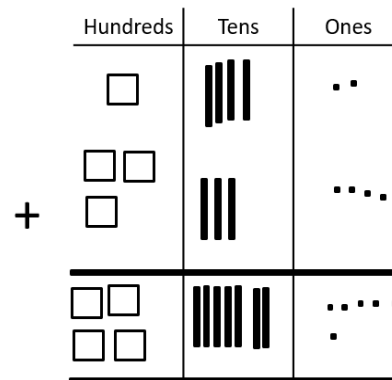
ADDITION

Formal written addition with no exchanging (up to three-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 142 \\ + 334 \\ \hline 476 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ The sum of ... tens and ... tens is ...tens.
- ❖ The sum of ... hundreds and ... hundreds is ... hundreds.
- ❖ So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

This step revisits the formal written method, first encountered in Year 2, with no exchanging but introduces hundreds.

Pupils should be encouraged to estimate first and check their answer using a mental method.

YEAR 3

NC Statement:

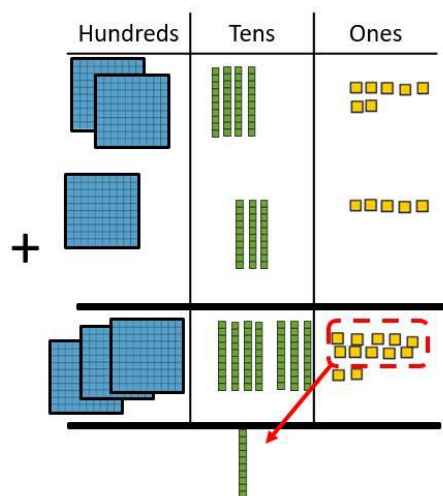
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



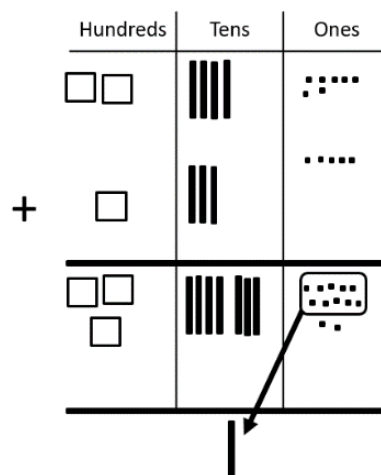
ADDITION

Formal written addition with exchanging of ones (up to three-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 247 \\ + 135 \\ \hline 382 \\ \hline 1 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ The sum of ... tens and ... tens is ...tens.
- ❖ The sum of ... hundreds and ... hundreds is ... hundreds.
- ❖ So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

The focus is on exchanging ones.

Pupils should be encouraged to estimate first and check their answer using a mental method.

YEAR 3

NC Statement:

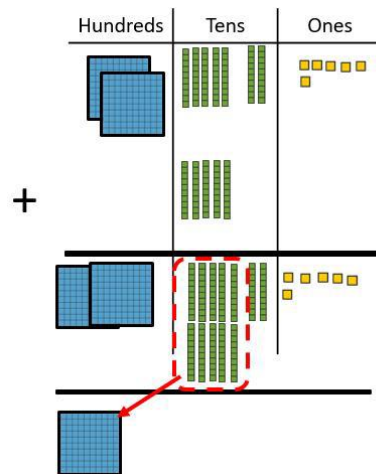
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



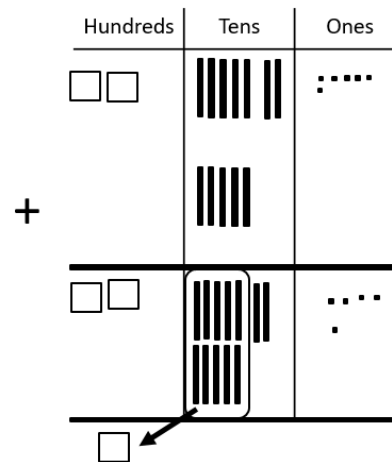
ADDITION

Formal written addition with exchanging tens only (up to three-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 276 \\ + 50 \\ \hline 326 \\ \hline 1 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ This is exchanged into ... tens and ... ones.
- ❖ The sum of ... tens and ... tens is ...tens.
- ❖ The sum of ... hundreds and ... hundreds is ... hundreds.
- ❖ So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

The focus is on exchanging of tens.

Pupils should be encouraged to estimate first and check their answer using a mental method.

YEAR 3

NC Statement:

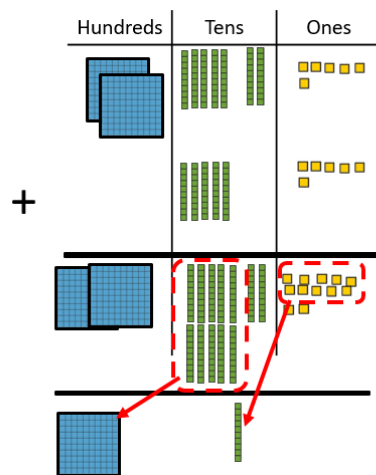
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



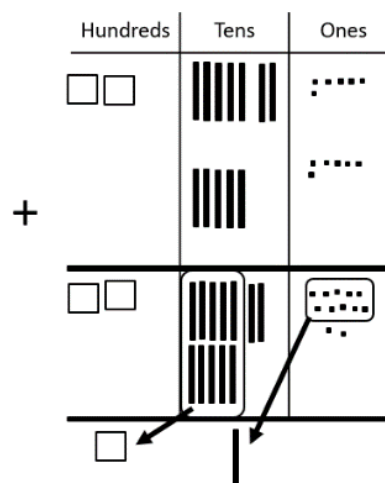
ADDITION

Formal written addition with exchanging tens and ones (up to three-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r}
 276 \\
 + 56 \\
 \hline
 332 \\
 \hline
 11
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ The sum of ... ones and ... ones is ... ones.
- ❖ This is exchanged into ... tens and ... ones.
- ❖ The sum of ... tens and ... tens is ...tens.
- ❖ This is exchanged into ... hundreds and ... tens.
- ❖ The sum of ... hundreds and ... hundreds is ... hundreds.
- ❖ So, ... + ... is equal to ... hundreds, ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

Once pupils have fully understood and rehearsed exchanging within formal column addition of 3-digit numbers, this learning continues to be rehearsed and applied throughout Years 4, 5 and 6, including to 4-digit numbers, larger numbers, decimal numbers, money and measures.

YEAR 2

NC Statement:

Add and Subtract numbers using concrete objects, pictorial representations, and mentally including:

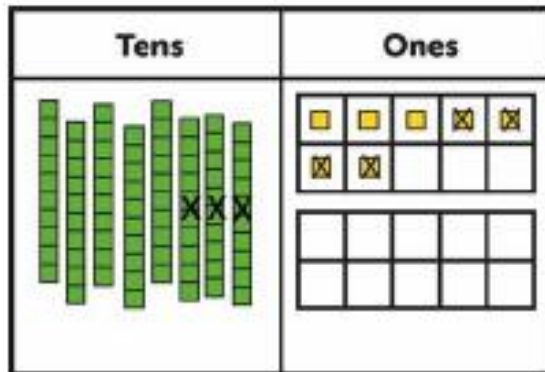
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers.



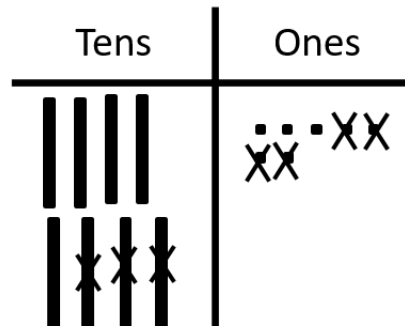
SUBTRACTION

Expanded written subtraction, a 2-digit number from a 2-digit number with no exchanging.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 80 + 7 \\ - 30 + 4 \\ \hline 50 + 3 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... ones take away ... ones leaves ... ones.
- ❖ ... tens take away ... tens leaves ... tens.
- ❖ So, ... - ... is equal to ... tens and ... ones, which is

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

YEAR 2

NC Statement:

Add and Subtract numbers using concrete objects, pictorial representations, and mentally including:

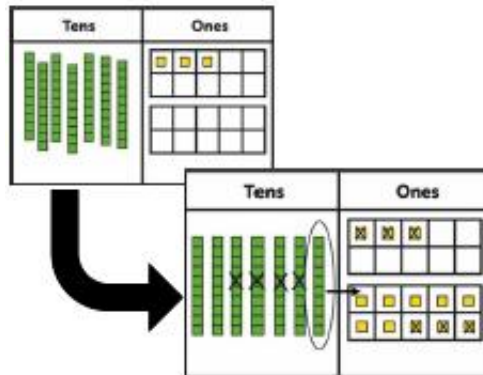
- a two-digit number and ones
- a two-digit number and tens
- two, two-digit numbers.



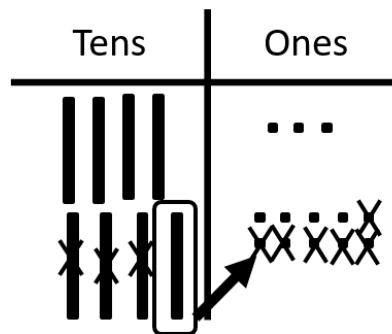
SUBTRACTION

Expanded written subtraction, a 2-digit number from a 2-digit number with exchanging.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 60 \quad 13 \\ 70 + 3 \\ - 40 + 6 \\ \hline 20 + 7 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I can see that there aren't enough ones for me to take away ... ones without exchanging. Exchange one ten into ten ones. There are now ... tens and ... ones.
- ❖ ... ones take away ... ones leaves ... ones.
- ❖ ... tens take away ... tens leaves ... tens.
- ❖ So, ... - ... is equal to ... tens and ... ones, which is

Notes:

Using embedded tens frame supports pupils to exchange accurately and to keep their jottings organised.

YEAR 3

NC Statement:

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



SUBTRACTION

Formal written subtraction with no exchanging (up to 3-digit numbers).

CONCRETE:

Hundreds	Tens	Ones

PICTORIAL:

Hundreds	Tens	Ones

ABSTRACT: WRITTEN

$$\begin{array}{r} 345 \\ - 124 \\ \hline 221 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... ones take away ... ones leaves ... ones.
- ❖ ... tens take away ... tens leaves ... tens.
- ❖ ... hundreds take away ... hundreds leaves ... hundreds.
- ❖ So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is ...

Notes:

Pupils should be encouraged to estimate first and check their answer using a mental method.

YEAR 3

NC Statement:

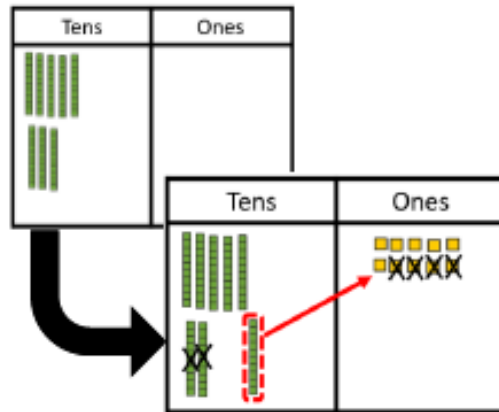
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



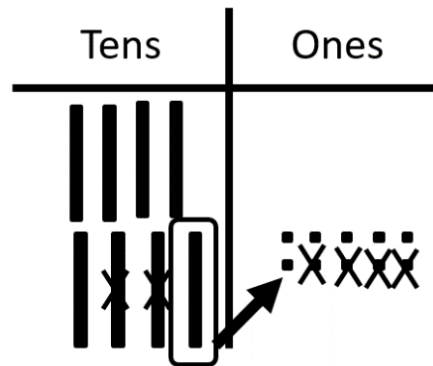
SUBTRACTION

Formal written subtraction - exchanging tens into ones only (up to 3-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 7\cancel{8}^{10} \\ - 24 \\ \hline 56 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I can see that there aren't enough ones for me to take away ... ones without exchanging. Exchange one ten into ten ones. There are now ... tens and ... ones.
- ❖ ... ones take away ... ones leaves ... ones.
- ❖ ... tens take away ... tens leaves ... tens.
- ❖ So, ... - ... is equal to ... tens and ... ones, which is

Notes:

It is important that pupils understand that 80 has been partitioned into 70 and 10. If pupils struggle with the compact notation refer to Year 2 'Expanded Written Method with exchanging'.

YEAR 3

NC Statement:

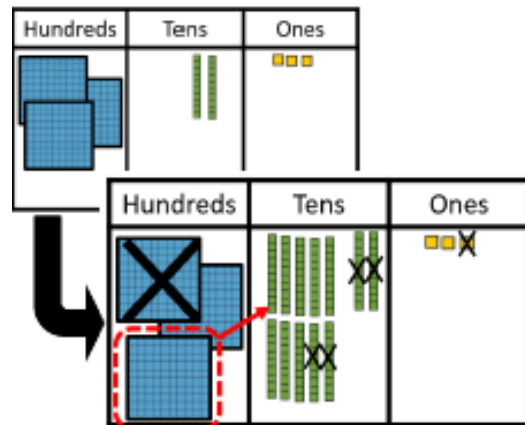
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.



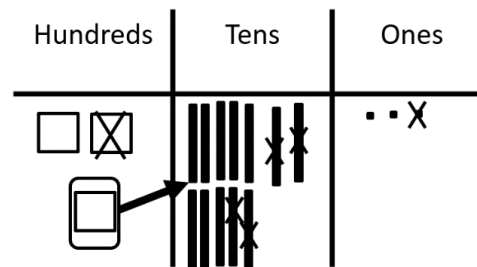
SUBTRACTION

Formal written subtraction - exchanging hundreds into tens only (up to 3-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r}
 \cancel{2}^2 \cancel{0}^1 2 \ 3 \\
 - 1 \ 4 \ 1 \\
 \hline
 1 \ 8 \ 2
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I can see that there aren't enough ones for me to take away ... ones without exchanging. Exchange one ten into ten ones. There are now ... tens and ... ones.
- ❖ ... ones take away ... ones leaves ... ones.
- ❖ ... tens take away ... tens leaves ... tens.
- ❖ So, ... - ... is equal to ... hundreds, ... tens and ... ones, which is

Notes:

It is important that pupils start to identify where exchanging is necessary. Ensure that pupils are confident that the minuend may have been exchanged but it is still of equal value prior to subtraction.

YEAR 3

NC Statement:

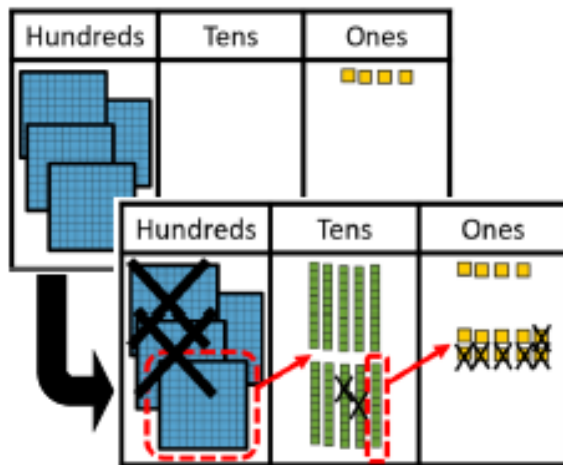
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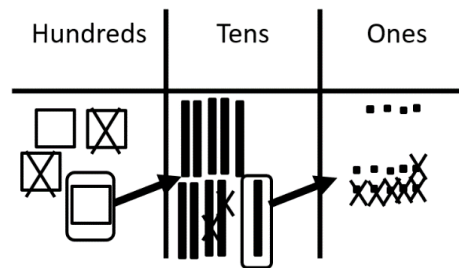
SUBTRACTION

Formal written subtraction - exchanging hundreds and tens (up to 3-digit numbers).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r}
 3 \cancel{0} 9 4 \\
 - 2 2 6 \\
 \hline
 1 7 8
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I will need to exchange...
- one hundred into ten tens. I now have ... hundreds and ... tens.
- one ten into ten ones. I now have ... tens and ... ones.

Notes:

Once pupils have fully understood and rehearsed exchanging within formal subtraction, this learning continues to be rehearsed and applied throughout Years 4, 5 and 6, including to multi-digit, decimal numbers, money and measures.

YEAR 3

NC Statement:

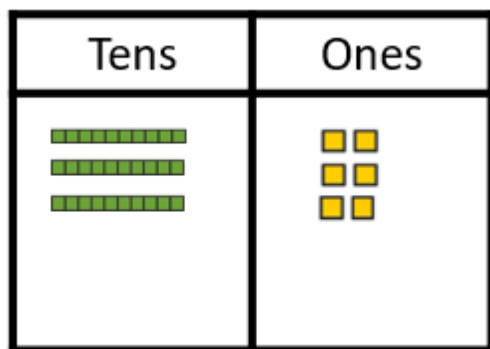
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



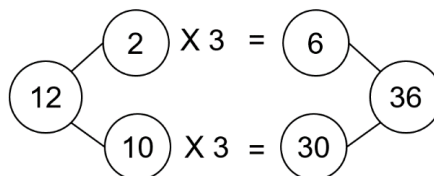
MULTIPLICATION

Introducing short multiplication with no exchanging.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 12 \\ \times 3 \\ \hline 36 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... groups of ... ones is ... ones.
- ... groups of ... tens is ... tens.
- ... tens added to ... ones is ...
- The product of ... and ... is ...

Notes:

Pupils have already met the distributive law and rehearsed multiplying by ten.

YEAR 3

NC Statement:

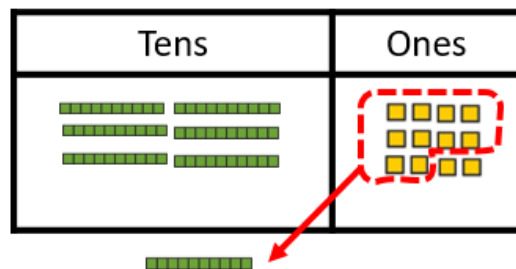
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



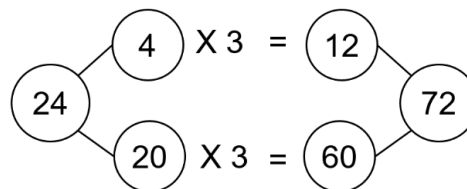
MULTIPLICATION

Short multiplication with exchanging of ones into tens only.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline 1 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... groups of ... ones is ... ones.
I can exchange the ... ones into ... ten(s) and ... one(s).
- ... groups of ... tens is ... tens.
- ... ten(s) added to ... is
- The product of ... and ... is

Notes:

Pupils have already met the distributive law and rehearsed multiplying by ten.

YEAR 3

NC Statement:

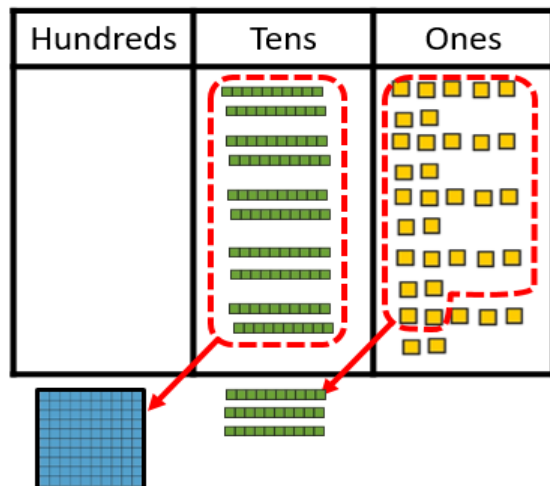
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



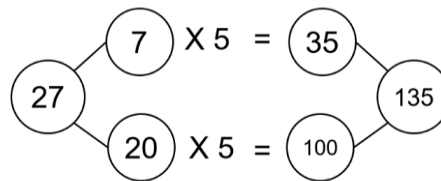
MULTIPLICATION

Short multiplication with exchanging of tens and ones.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 27 \\ \times 5 \\ \hline 135 \\ \hline 3 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... groups of ... ones is ... ones.
- I can exchange the ... ones into ... ten(s) and ... one(s).
- ... groups of ... tens is ... tens.
- ... ten(s) added to ... ten(s) is ..
- I can exchange the ... tens into ... hundred(s) and ... ten(s).
- The product of ... and ... is ...

Notes:

At this stage, the pictorial representation is being used as a checking point to ensure pupils answer accurately. This allows focused attention on understanding the abstract recording.

YEAR 4

NC Statement:

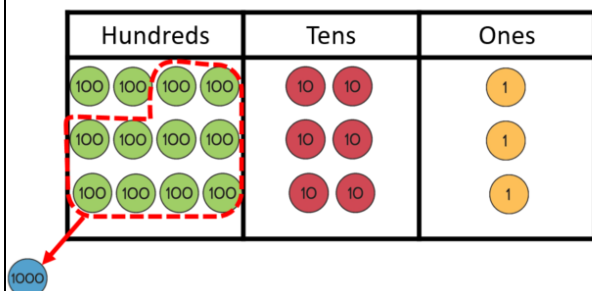
Multiply 2-digit and 3-digit numbers by a one-digit number using formal written layout (short multiplication).



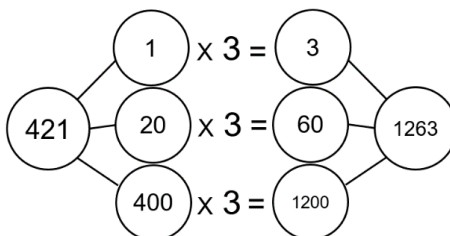
MULTIPLICATION

Formal written multiplication with exchanging which generates a new column.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 421 \\ \times \quad 3 \\ \hline 1263 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ ... groups of ... ones is ... ones. (Do I need to exchange?)
- ... groups of ... tens is ... tens. (Do I need to exchange?)
- ... groups of ... hundreds is ... hundreds. (Do I need to exchange?)
- (... hundreds can be exchanged to ... thousands and ... hundreds)
- The product of ... and ... is

Notes:

At this stage, the pictorial representation is being used as a checking point to ensure that pupils answer accurately. This allows focused attention on understanding the abstract recording.

YEAR 5

NC Statement:

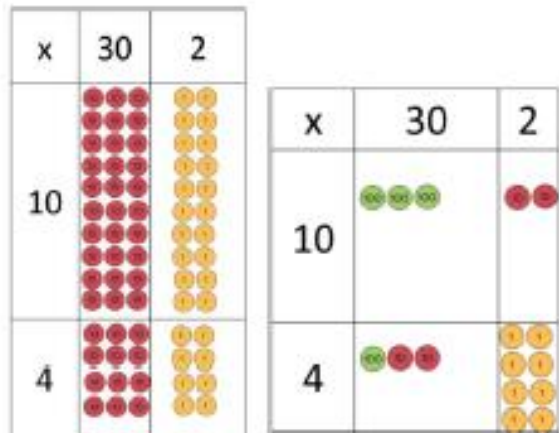
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.



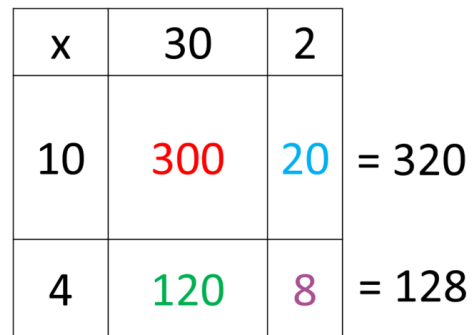
MULTIPLICATION

Expanded long multiplication 2-digit by 2-digit.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r}
 32 \\
 \times 14 \\
 \hline
 8 \\
 120 \\
 300 \\
 \hline
 448
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

❖ First, I need to consider the ones in the multiplier.

... groups of ... ones is ones.

... groups of ... tens is tens. (Do I need to exchange?)

Then, tens in the multiplier.

... groups of ... ones is ones. (Do I need to exchange?)

... groups of ... tens is tens. (Do I need to exchange?)

The total of all the partial products is ...

The product of ... and ... is ...

Notes:

This is a transitional method towards long multiplication. Using the grid supports pupils in their thinking about multiplying by powers of ten and place value. Secure understanding of both of these concepts allow pupils to move to long multiplication more successfully.

YEAR 5

NC Statement:

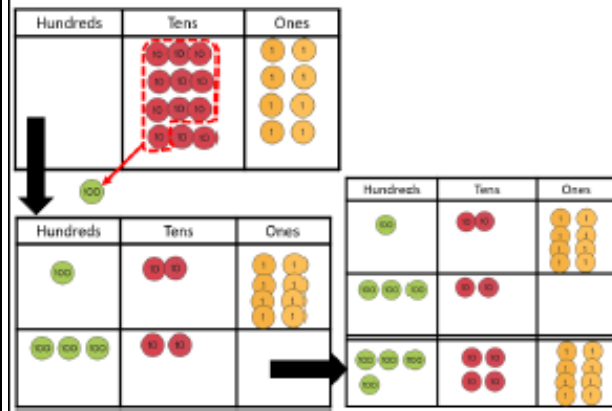
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.



MULTIPLICATION

Long multiplication 2-digit by 2-digit with simple exchanging.

CONCRETE:



PICTORIAL:

x	30	2	
10	300	20	= 320
4	120	8	= 128

ABSTRACT: WRITTEN

$$\begin{array}{r}
 32 \\
 \times 14 \\
 \hline
 128 \\
 320 \\
 \hline
 448
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ First, I need to consider the ones in the multiplier.
- ... groups of ... ones is ones. (Do I need to exchange?)
- ... groups of ... tens is tens. (Do I need to exchange?)
- Then, considering tens in the multiplier.
- ... groups of ... ones is ones. (Do I need to exchange?)
- ... groups of ... tens is tens. (Do I need to exchange?)
- The total of all the partial products is ...
- The product of ... and ... is ...

Notes:

YEAR 5

NC Statement:

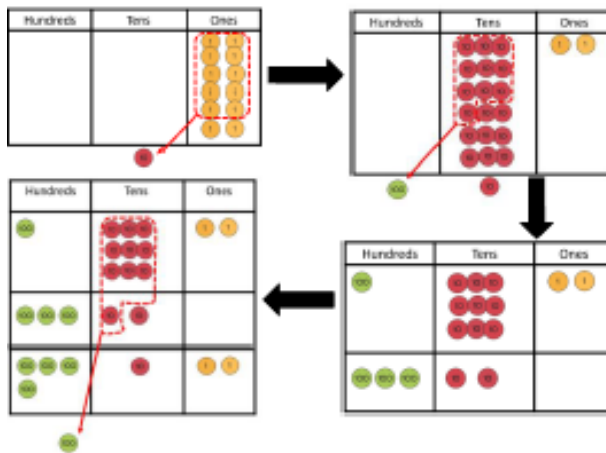
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.



MULTIPLICATION

Long multiplication 2-digit by 2-digit, focusing on exchange in first partial product.

CONCRETE:



PICTORIAL:

x	30	2	
10	300	20	= 320
6	180	12	= 192

ABSTRACT: WRITTEN

$$\begin{array}{r}
 32 \\
 \times 16 \\
 \hline
 192 \\
 320 \\
 \hline
 512
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

❖ First, I need to consider the ones in the multiplier.

... groups of ... ones is ... ones. (Do I need to exchange?)
 ... groups of ... tens is ... tens. (Any exchanges to add?)
 Do I need to exchange?)

Then, considering tens in the multiplier.

... groups of ... ones is ... ones. (Do I need to exchange?)
 ... groups of ... tens is ... tens. (Do I need to exchange?)

The total of all the partial products is ...

Notes:

YEAR 5

NC Statement:

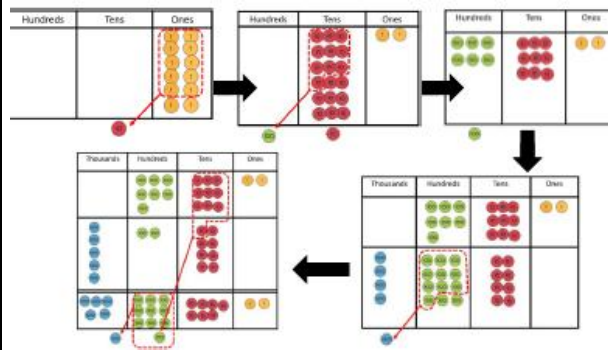
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.



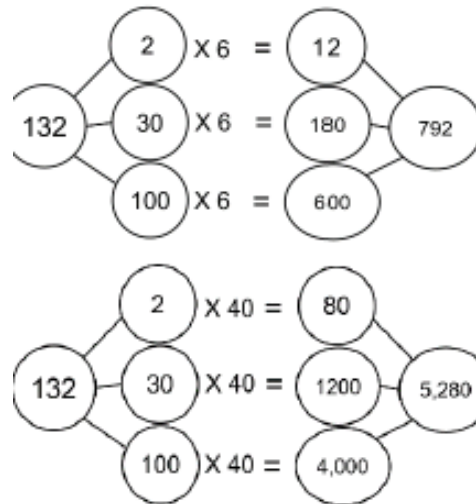
MULTIPLICATION

Long multiplication 2-digit by 2-digit exchanging in first and second stage.

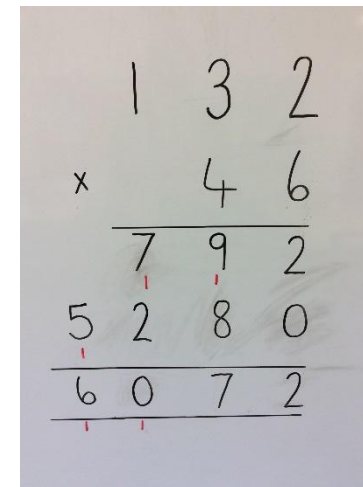
CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN



ABSTRACT - SPEAKING FRAME:

- ❖ First, I need to consider the ones in the multiplier.
- ... groups of ... ones is ... ones. (Do I need to exchange?)
- ... groups of ... tens is ... tens. (Any exchanges to add? Do I need to exchange?)
- Then, considering tens in the multiplier.
- ... groups of ... ones is ... ones. (Do I need to exchange?)
- ... groups of ... tens is ... tens. (Any exchanges to add? Do I need to exchange?)
- The total of all the partial products is

Notes:

Once pupils have fully understood and rehearsed exchanging within formal multiplication, this learning continues to be rehearsed and applied throughout Year 6 also, including to multi-digit, decimal numbers, money and measures.

YEAR 6

NC Statement:

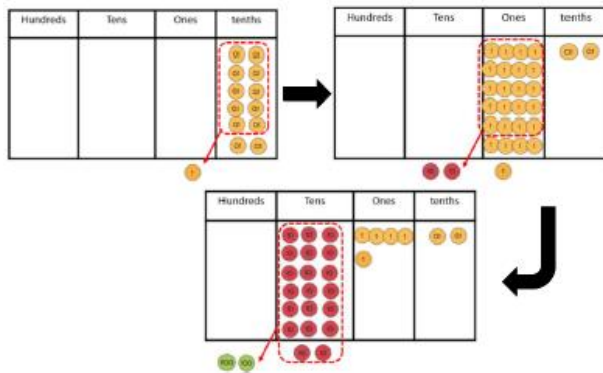
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.



MULTIPLICATION

Formal written multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number.

CONCRETE:



PICTORIAL:

Jottings: multiples of tricky multipliers

6
12
18
24
30
36
42
48
54
60
66
72

ABSTRACT: WRITTEN

A photograph of a handwritten multiplication problem on a whiteboard. The problem is 34.2 multiplied by 6. The student has written the product as 205.2. Red lines and numbers are drawn under the product to show the carrying process: a '2' is written under the 0, and a '1' is written under the 5.

ABSTRACT - SPEAKING FRAME:

- ❖ ... groups of ... tenths is ... tenths. (Do I need to exchange?)
- ... groups of ... ones is ... ones. (Any exchanges to add? Do I need to exchange?)
- ... groups of ... tens is ... tens. (Any exchanges to add? Do I need to exchange?)
- The product of ... and ... is

Notes:

YEAR 3

NC Statement:

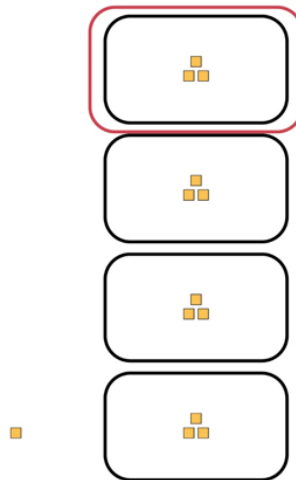
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



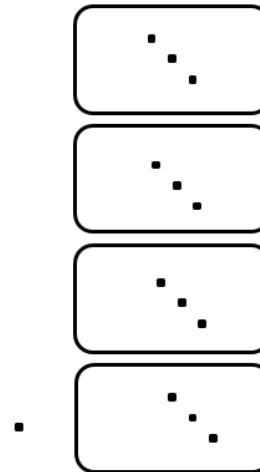
DIVISION

Introducing the long division method (sharing ones).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 3 \\ 4 \overline{) 13} \\ \underline{- 12} \\ 1 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I am sharing ... ones into ... equal groups.
- ❖ There are ... ones in each group.
- ❖ I have ... one(s) remaining.
- ❖ The quotient is ... with ... remainders.

Notes:

Pupils are introduced to the long division method for the first time in this sequence. Short division will not be introduced until pupils have understood all of the stages in this expanded form. In the calculation $96 \div 4$, for example, pupils often struggle to understand that 1 ten will be exchanged after 8 tens have been used in the 4 groups. This is hidden in short division but recorded in long division.

Vocabulary:

Dividend - The number being divided into pieces.

Divisor - The number by which the dividend is divided.

Quotient - The answer to the division problem.

YEAR 3

NC Statement:

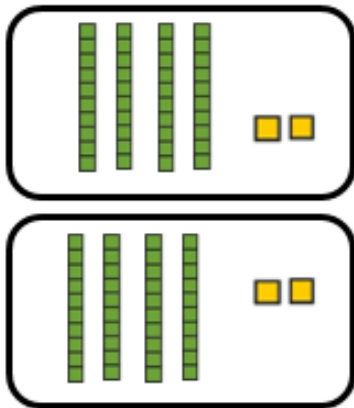
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



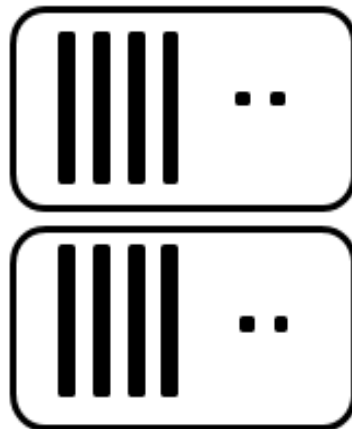
DIVISION

Long division of tens and ones with no exchanging (sharing structure).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 42 \\ 2 \overline{) 84} \\ \underline{- 8} \\ 04 \\ \underline{- 4} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ First, I am sharing ... tens into ... equal groups.
- ❖ There are ... tens in each group.
- ❖ I have ... ten(s) remaining.
- ❖ Then, I am sharing ... ones into ... equal groups.
- ❖ There are ... ones in each group.
- ❖ I have ... one(s) remaining.
- ❖ The quotient is ... with ... remainders.

Notes:

This stage is to support pupils' understanding of the abstract notation. They learn to record how many tens are in each group, if there are any tens remaining and what the arrow means.

YEAR 3

NC Statement:

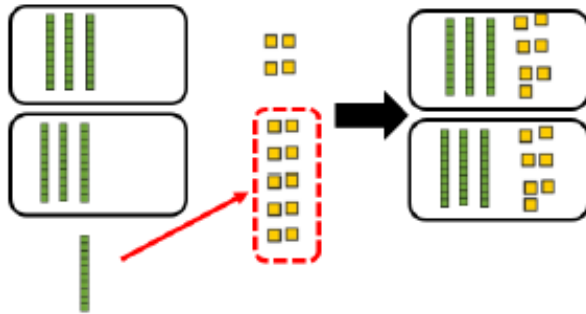
Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.



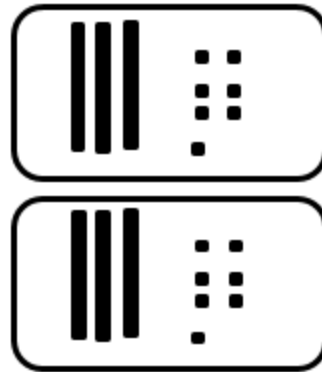
DIVISION

Long division of tens and ones with exchanging (sharing structure).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 37 \\ 2 \overline{) 74} \\ \underline{- 6} \\ 14 \\ \underline{- 14} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ First, I am sharing ... tens into ... equal groups.
- ❖ There are ... tens in each group.
- ❖ I have ... ten(s) remaining.
- ❖ I need to exchange the remaining ... ten(s) into ... ones.
- ❖ I now have ... ones in total.
- ❖ Then, I am sharing ... ones into ... equal groups.
- ❖ There are ... ones in each group.
- ❖ I have ... one(s) remaining.
- ❖ The quotient is ... with ... remainders.

Notes:

This is a crucial stage as it demonstrates the exchanging of the remaining tens for ones and how this is recorded abstractly.

YEAR 4

NC Statement:

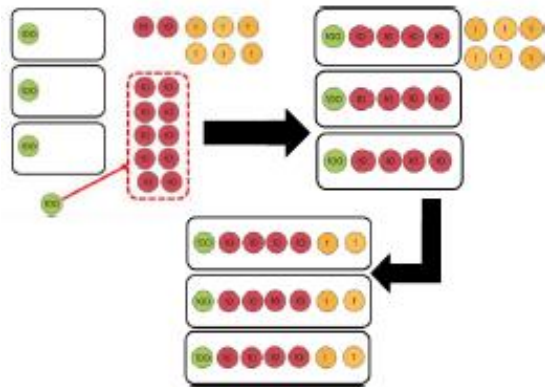
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (*non-statutory guidance*).



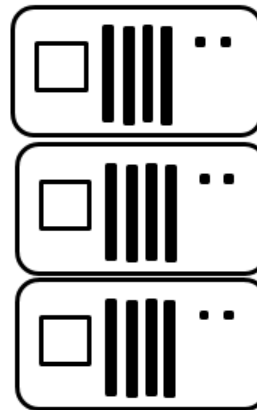
DIVISION

Long division with exchanging hundreds into tens (*sharing structure*).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 142 \\ 3 \overline{) 426} \\ \underline{- 3} \\ 12 \\ \underline{- 12} \\ 06 \\ \underline{- 06} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ First, I am sharing ... hundreds into ... equal groups.
- ❖ There are ... hundreds in each group.
- ❖ I have ... hundred(s) remaining.
- ❖ I need to exchange the remaining ... hundreds into ...tens.
- ❖ I now have ... tens in total.
- ❖ Next, I am sharing ... tens into .. equal groups.

Notes:

Pupils revisit long division with no exchanging. This is to ensure that they understand the abstract recording of long division.

YEAR 4

NC Statement:

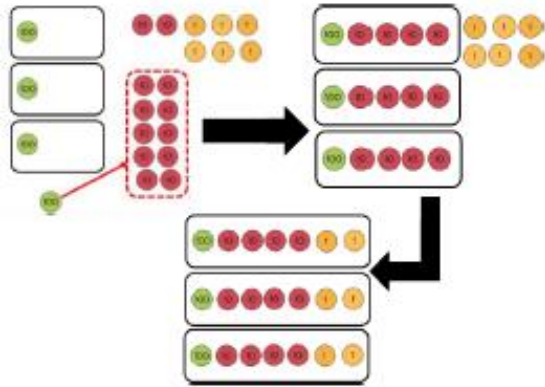
Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (*non-statutory guidance*).



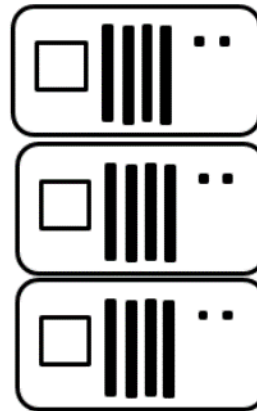
DIVISION

Introducing formal short division (sharing structure).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 142 \\ 3 \overline{) 426} \\ \underline{3} \\ 12 \\ \underline{9} \\ 36 \\ \underline{36} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ First, I am sharing ... hundreds into ... equal groups.
- ❖ There are ... hundreds in each group.
- ❖ I have ... hundred(s) remaining.
- ❖ I need to exchange the remaining ... hundreds into ... tens.
- ❖ I now have ... tens in total.
- ❖ Next, I am sharing ... tens into .. equal groups.

Notes:

In this stage, pupils learn that the thinking processes for long and short division are the same - it is only the abstract written that is different. It is important that pupils are able to link this to the long division format and can explain the compaction.

YEAR 5

NC Statement:

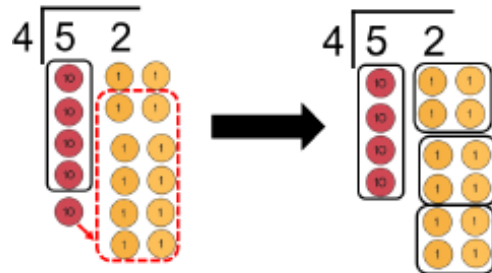
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



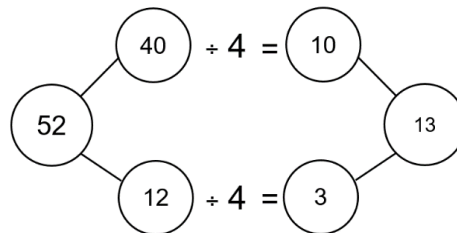
DIVISION

Introducing formal short division exchange from tens to ones (grouping structure).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$4 \overline{) 52} \begin{array}{r} 13 \\ \underline{4} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I want to know how many groups of ... are in ...
- ❖ How many groups of ... tens are in ... tens without exchanging?
- ❖ I can make ... group(s) of ... tens. There is/are ... ten(s) remaining.
- ❖ I need to exchange the ... tens into ... ones.
- ❖ I now have ... ones.
- ❖ How many groups of ... ones are in ... ones, without exchanging?
- ❖ I can make ... group(s) of ... ones. There is/are ... one(s) remaining.
- ❖ There are ... groups of ... in ... with ... remainders.

Notes:

Pupils are encouraged to progress to a grouping model of division. This is in preparation for 2-digit divisors and understanding fractions expressed as part of the quotient.
Pupils should explore with simple division calculations to ensure that they understand the shift in structure.

YEAR 5

NC Statement:

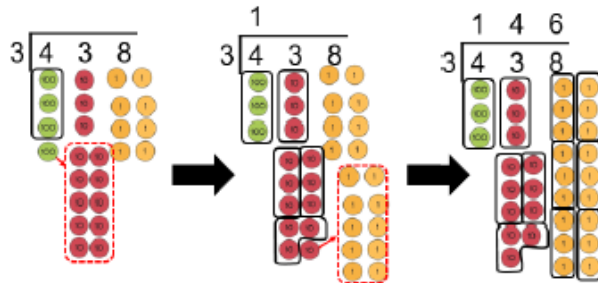
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



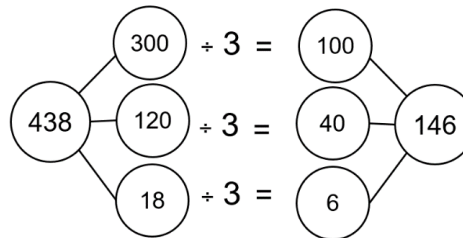
DIVISION

Short division for numbers up to 4-digits (grouping structure).

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$\begin{array}{r} 146 \\ 3 \overline{) 438} \\ \underline{3} \\ 13 \\ \underline{12} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I want to know how many groups of ... are in ...
- ❖ How many groups of ... hundreds are in ... hundreds, without exchanging?
- ❖ I can make ... group(s) of ...hundreds. There is/are ... hundred(s) remaining.
- ❖ I need to exchange the ... hundreds into ...tens.

Notes:

YEAR 5

NC Statement:

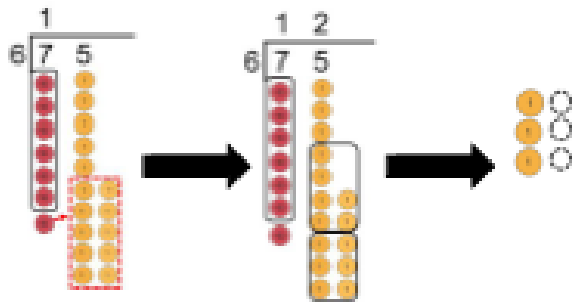
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



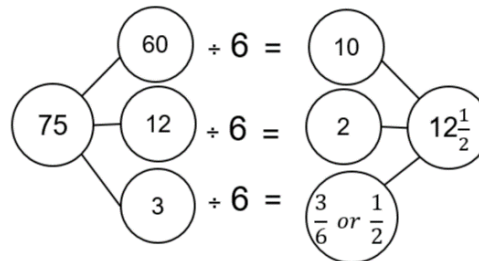
DIVISION

Short division (grouping structure) - expressing quotients with fractions.

CONCRETE:



PICTORIAL:



ABSTRACT: WRITTEN

$$6 \overline{) 75} = 12 \frac{1}{2}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I have a remainder of
- ❖ This is ... (remainder) out of ... (divisor) which I need for another group.
- ❖ This can be written as a fraction .
- ❖ This can be simplified to .

Notes:

YEAR 5

NC Statement:

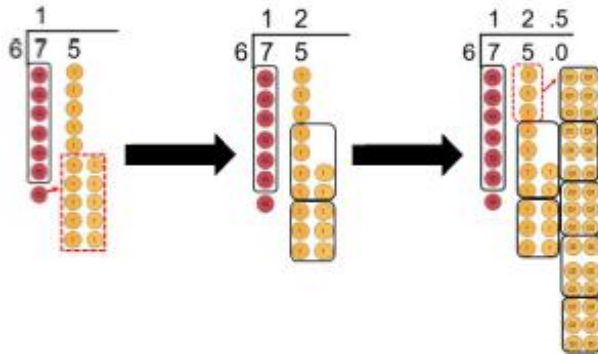
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.



DIVISION

Short division (grouping structure) - expressing quotients with decimals.

CONCRETE:



PICTORIAL:

Jottings: multiples of the divisor

6
12
18
24
30
36
42
48
54
60

ABSTRACT: WRITTEN

$$\begin{array}{r} 12.5 \\ 6 \overline{) 75.0} \\ \underline{6} \\ 15 \\ \underline{12} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I have a remainder of
- ❖ I need to exchange the ... ones into ... tenths.
- ❖ How many groups of ... tenths are in ... tenths, without exchanging?
- ❖ I can make ... group(s) of ... tenths.
- ❖ There are ... groups of ... in

Notes:

YEAR 6

NC Statement:

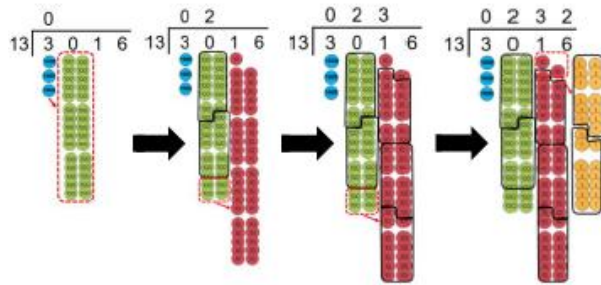
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.



DIVISION

Long division for numbers up to 4 digits.

CONCRETE:



PICTORIAL:

Jottings: multiples of the divisor

13
26
39
52
65
78
91
104

ABSTRACT: WRITTEN

$$\begin{array}{r}
 \\
 13 \overline{) 3016} \\
 \underline{- 0} \\
 30 \\
 \underline{- 26} \\
 41 \\
 \underline{- 39} \\
 26 \\
 \underline{- 26} \\
 0
 \end{array}$$

ABSTRACT - SPEAKING FRAME:

- ❖ I want to know how many groups of ... are in ...
- ❖ How many groups of ... thousand are in ...thousand, without exchanging?
- ❖ I can make ... group(s) of ...thousand. There is/are ... thousand(s) remaining.
- ❖ I need to exchange the ... thousand(s) into ...hundreds.

Notes:

The structure of long division was first introduced in Year 3 then revisited and extended in both Years 4 and 5. It was revised in Step 1 of this sequence. Jottings are used to scaffold to derived related division facts.

YEAR 6

NC Statement:

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.



DIVISION

Long division for numbers up to 4 digits - expressing quotients with fractions.

ABSTRACT: WRITTEN

$$\begin{array}{r} 037\frac{3}{5} \\ 15 \overline{) 564} \\ \underline{- 0} \\ 56 \\ \underline{- 45} \\ 114 \\ \underline{- 105} \\ 9 \end{array}$$

$\frac{9}{15} = \frac{3}{5}$

Long division for numbers up to 4 digits - expressing quotients with decimals.

ABSTRACT: WRITTEN

$$\begin{array}{r} 037.6 \\ 15 \overline{) 564.0} \\ \underline{- 0} \\ 56 \\ \underline{- 45} \\ 114 \\ \underline{- 105} \\ 90 \\ \underline{- 90} \\ 0 \end{array}$$